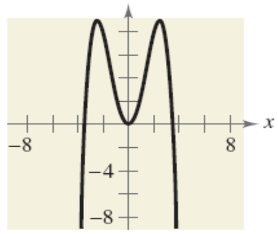
Honors Alg2/Trig Unit 5 Higher Order Polynomials Review Name:____________________

Part One: End Behavior, Roots, Graphing

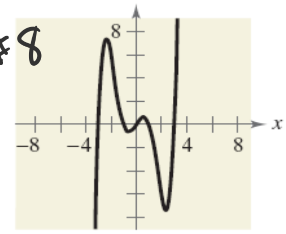
Match the polynomial function with its graph.

1. $f(x) = -2x + 3$ $\boxed{C}$
2. $f(x) = x^2 - 4x$ $\boxed{g}$
3. $f(x) = -2x^2 - 5x$ $\boxed{h}$
4. $f(x) = 2x^3 - 3x + 1$ $\boxed{F}$
5. $f(x) = -\frac{1}{4}x^4 + 3x^2$ $\boxed{A}$
6. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$ $\boxed{E}$
7. $f(x) = x^4 + 2x^3$ $\boxed{D}$
8. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$ $\boxed{B}$

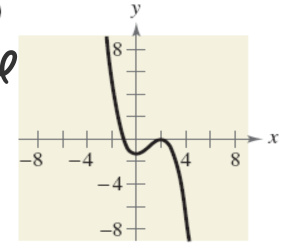
(a) #5



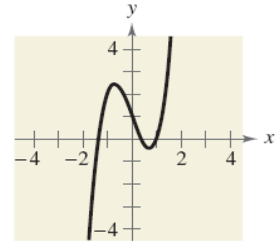
(b) #8



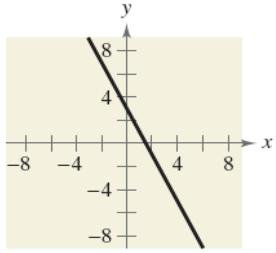
(e) #6



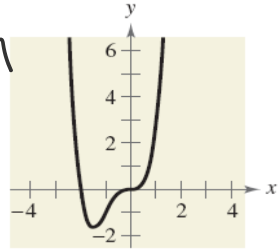
(f) #4



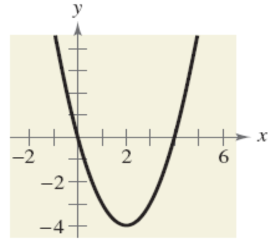
(c) #1



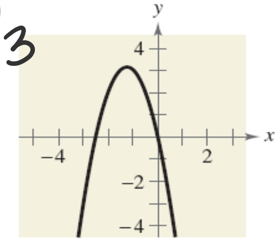
(d) #7



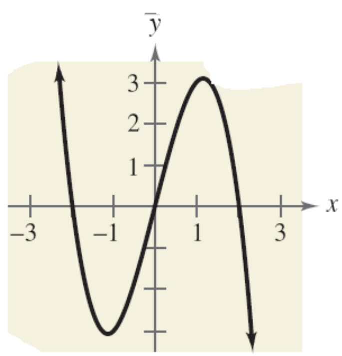
(g) #2



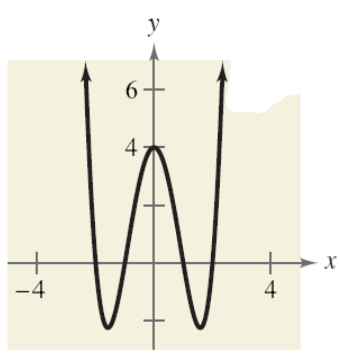
(h) #3



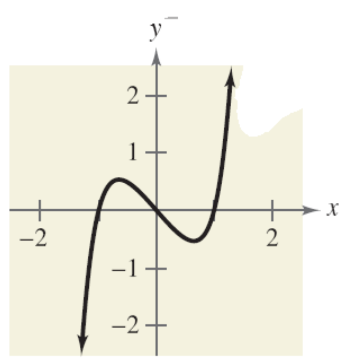
Identify the roots of the graphs below.



Roots: -2, 0, 2



Roots: -2, -1, 1, 2

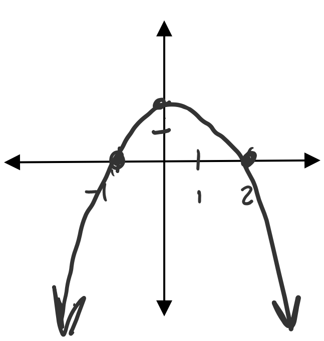


Roots: -1, 0, 1

Write the polynomial equation (in factored form) with the following roots and leading coefficients. Draw a rough sketch of the graph.

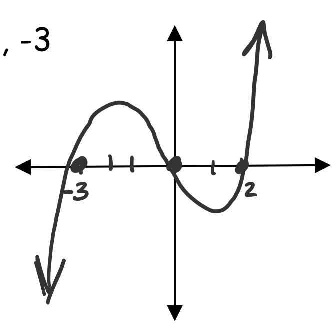
1) a < 0 , x=2, -1

$-a(x-2)(x+1)$

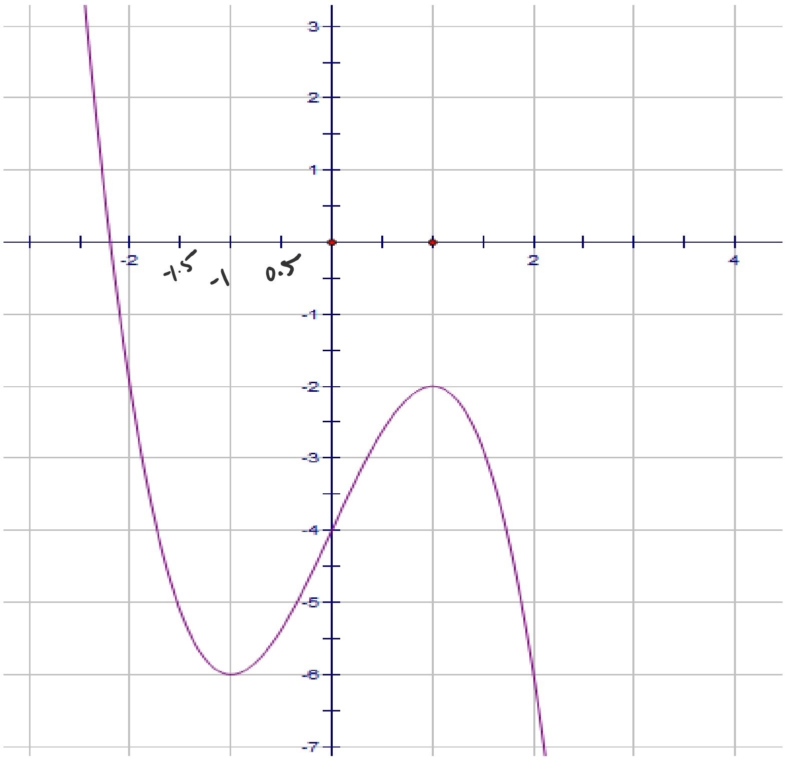


2) a > 0 , x = 0 (triple root), 2, -3

$x^3(x-2)(x+3)$



Fill in the following information for the graph below.



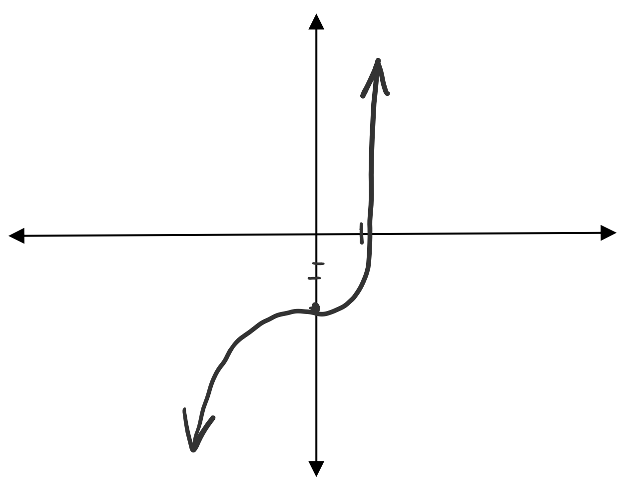
Zeros: -0.25, 2 imag.
Relative Maximum: $x=1$
Relative Minimum: $x=-1$
Increasing: $(-1, 1)$
Decreasing: $(-\infty,-1)\cup(1,\infty)$

Use your graphing calculator to find the following information. Then, sketch the graph.

$f(x) = x^3 + 2x^2 - 3$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & 0 & -3 \\ & & 1 & 3 & 3 \\ \hline & 1 & 3 & 3 & 10 \end{array}$$

Zeros: $x=1,\ x=\frac{-3\pm i\sqrt{3}}{3}$
Relative Maximum: $x=-1.\overline{33}$
Relative Minimum: $x=0$
Increasing: $(-\infty,-1.\overline{33})\cup(0,\infty)$
Decreasing: $(-1.\overline{33},0)$



$$x^2+3x+3=0$$

$$x=\frac{-3\pm\sqrt{3^2-4(1)(3)}}{2(1)}=\frac{-3\pm\sqrt{9-12}}{3}$$

$$=\frac{-3\pm\sqrt{-3}}{3}$$

Determine the number of roots for each function below:

a) $f(x) = -2x - 2$ 1

b) $f(x) = x^3 - 2x^2 + 5$ 3

c) $f(x) = x^4 + 4x^3 - 22x + 1$ 4

Part Two: Finding Roots Algebraically, Synthetic Division, Rational Root Theorem

Find the roots/zeros of the following polynomials by using any algebraic method. Identify what kinds of roots you find.

1) $f(x) = x^4 + 4x^2 - 5$

$(x^2+5)(x^2-1)=0$

$x^2=-5$ $\quad (x-1)(x+1)=0$

$\boxed{x=\pm i\sqrt{5}}$ $\boxed{x=1}$ $\boxed{x=-1}$

3) $f(x) = -4x^3 + 4x^2 + 15x$

$-x(4x^2-4x-15)$ $\quad x^2-4x-60$

$-x(2x+3)(2x-5)$ $\quad (x+\frac{6}{4})(x-\frac{10}{4})$

$\boxed{x=0}$ $\boxed{x=-3/2}$ $\boxed{x=5/2}$

2) $f(x) = 2x^4 - 2$

$2(x^4-1)$

$2(x^2-1)(x^2+1)$

$\boxed{x=\pm 1}$ $\boxed{x=\pm i}$

4) $f(x) = 3x^3 - 2x^2 - 9x + 6$

$x^2(3x-2)-3(3x-2)$

$(x^2-3)(3x-2)$

$\boxed{x=\pm\sqrt{3}}$ $\boxed{x=2/3}$

Divide using synthetic division. Then, find the remaining roots.

a) $f(x) = x^3 - 9x^2 + 27x - 27$ by $(x-3)$

$$\begin{array}{c|cccc} 3 & 1 & -9 & 27 & -27 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$\boxed{x=3 \quad T.R.}$

$(x-3)(x^2-6x+9)$

$(x-3)(x-3)(x-3)$

b) $f(x) = 4x^3 - 8x^2 + x + 3$ by $(x-1)$

$$\begin{array}{c|cccc} 1 & 4 & -8 & 1 & 3 \\ & & 4 & -4 & -3 \\ \hline & 4 & -4 & -3 & 0 \end{array}$$

$\boxed{x=1}$ $\boxed{x=\frac{3}{2}}$ $\boxed{x=-1/2}$

$(x-1)(4x^2-4x-3)=0$

$x^2-4x-12$

$(x-\frac{6}{4})(x+\frac{2}{4})$

Determine if (x-7) is a factor of $f(x) = x^3 - 2x^2 - 30x - 35$. Show all work. Explain in one sentence why or why not (x-7) is a factor.

$$\begin{array}{r|rrrr} 7 & 1 & -2 & -30 & -35 \\ & & 7 & 35 & 35 \\ \hline & 1 & 5 & 5 & 0 \end{array}$$

yes (x-7) is a factor!

Find the roots/zeros of the following polynomials by using a graphing calculator.

$f(x) = -2x^3 + 5x^2 + 9x - 8$

$x = -1.67$ $\quad x = 0.70$ $\quad x = 3.47$

If x-3 is a factor of f(x) and $f(x) = x^3 - 8x^2 + kx + 42$, What is the value of k?

$f(3) = 0$

$0 = 3^3 - 8(3)^2 + k(3) + 42$

$0 = 27 - 72 + 3k + 42$

$0 = -3 + 3k$

$3 = 3k$

$\boxed{k = 1}$

List all the possible rational roots (p/q). Then find the roots/zeros of the following polynomials by using any algebraic method. Then, sketch a graph of the function.

1) $f(x) = 2x^3 + 13x^2 - 13x + 3$

$\dfrac{p: \pm 3, 1}{q: \pm 2, 1}$ $\boxed{\pm 3, 1, \tfrac{3}{2}, \tfrac{1}{2}}$

$$\begin{array}{r|rrrr} \tfrac{1}{2} & 2 & 13 & -13 & 3 \\ & & 1 & 7 & -3 \\ \hline & 2 & 14 & -6 & 0 \end{array}$$

$(2x-1)(2x^2 + 14x - 6)$

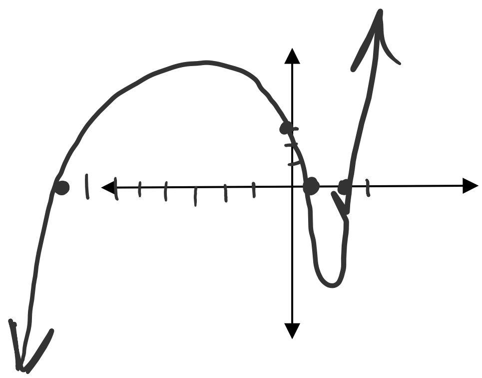
$2(x^2 + 7x - 3)$

$x = \dfrac{-7 \pm \sqrt{7^2 - 4(2)(-3)}}{2(1)}$

$\boxed{x = \tfrac{1}{2}}$ $\quad x = \dfrac{-7 \pm \sqrt{73}}{2}$

$\boxed{x = 0.77}$

$\boxed{x = -7.77}$



2) $f(x) = x^4 + 6x^3 - x^2 - 54x - 72$

$\pm\dfrac{p: 72, 36, 24, 18, 12, 9, 8, 6, 4, 3, 2, 1}{q: 1}$

$$\begin{array}{r|rrrrr} -4 & 1 & 6 & -1 & -54 & -72 \\ & & -4 & -8 & 36 & 72 \\ \hline & 1 & 2 & -9 & -18 & 0 \end{array}$$

$(x^3 + 2x^2)(-9x - 18)$

$x^2(x+2) - 9(x+2)$

$(x^2 - 9)(x+2)(x+4)$

$\boxed{x = 3}$ $\boxed{x = -3}$ $\boxed{x = -2}$ $\boxed{x = -4}$